

The Fractal Properties of the Source and BEC *

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Abstract

Using simple space-time implementation of the random cascade model we investigate numerically influence of the possible fractal structure of the emitting source on Bose-Einstein correlations between identical particles. The results are then discussed in terms of the non-extensive Tsallis statistics.

1 Formulation of the problem

1.1 Introduction

Two features seen in the analysis of multiparticle spectra of secondaries produced in high energy collision processes are of particular interest: (i) intermittent behaviour observed in analysis of factorial moments and (ii) Bose-Einstein correlations (BEC) observed between identical particles. Whereas the former indicates a possible (multi) fractal structure of the production process (in the momentum space) [1] the latter provides us with knowledge on the space-time aspects of production processes [2]. It was argued [3] that these features are compatible with each other only when: (i) either the shape of interaction region is regular but its size fluctuates from event to event according to some power-like scaling law or (ii) the interaction region itself is a self-similar fractal (in coordinate space) extending over a very large volume. Although there exists a vast literature on the (multi)fractality in momentum space [4] its space-time aspects are not yet fully recognized with [3] remaining so far the only representative investigations in this field. We shall present here numerical analysis of a particle production model possessing both the momentum and coordinate space fractalities [5] extending therefore ideas discussed in [3] to a more realistic scenario.

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1.2 Cascade model used

As a model we shall choose a simple self-similar cascade process of the type discussed in [6] but developed further to meet our demands. The following point must be stressed. Every cascade model is expected to lead automatically to intermittent behaviour of momentum spectra of observed particles [7]. Although this is true for models based on random multiplicative processes in observed variables (like energy, rapidity or azimuthal angle), this is not necessarily the case for multiplicative processes in variables which are not directly measurable but which are, nevertheless, of great dynamical importance (like masses M_i of intermediate objects in a cascade process considered here). In a purely mathematical case, where cascade process proceeds *ad infinitum*, one eventually always arrives at some fractal picture of the production process. However, both the finite masses μ of produced secondaries and the limited energy M stored originally in the emitting source prevent the full development of such fractal structure [8]. One must be therefore satisfied with only limited and indirect presence of such structure. This applies also to the analysis presented here.

In our model some initial mass M “decays” into two masses, $M \longrightarrow M_1 + M_2$ with $M_{1,2} = k_{1,2} \cdot M$ and $k_1 + k_2 < 1$ (i.e., a part of M equal to $(1 - k_1 - k_2)M$ is transformed to kinetic energies of the decay products $M_{1,2}$). The process repeats itself until $M_{1,2} \geq \mu$ (μ being the mass of the produced particles) with successive branchings occurring sequentially and independently from each other, and with *a priori* different values of $k_{1,2}$ at each branching, but with energy-momentum conservation imposed at each step. For different choices of dimensionality D of cascade process, $D = 1$ (linear) or $D = 3$ (isotropic), and for different (mostly random) choices of decay parameters $k_{1,2}$ at each vertex, we are covering a variety of different possible production schemes, ranging from one-dimensional strings to isotropic three-dimensional thermal-like fireballs. For our purpose of investigation of connections between BEC and space-time fractality of the source we have extended this (momentum space) cascade also to the space-time and we have added to it a kind of BEC “afterburner” along the lines advocated recently in [9].

What concerns space-time development we model it by introducing a fictitious finite life time t for each vertex mass M_i , distributed according to some prescribed distribution law given by

$$\Gamma(t) = \frac{2-q}{\tau} \cdot \left[1 - (1-q) \frac{t}{\tau} \right]^{\frac{1}{1-q}} \quad |q-1| \rightarrow 0 \quad \Gamma(t) = \frac{1}{\tau} \cdot \exp \left[-\frac{t}{\tau} \right]. \quad (1)$$

This procedure is purely classical, i.e., intermediate masses M_i are not treated as resonances (as was done in [10]) but are regarded to be stable clusters with masses given by the corresponding values of decay parameters $k_{1,2}$ and with velocities $\vec{\beta} = \vec{P}_{1,2}/E_{1,2}$ ($(E_{1,2}; \vec{P}_{1,2})$ are the corresponding energy-momenta of decay products calculated in each vertex in the rest frame of the parent mass). The energy-momentum and charges are strictly conserved in each vertex separately. The form of Γ used in (1) allows to account for the possible fluctuations of the evolution parameter τ [11] and finds its justification in the Tsallis statistics, to which we shall return later when discussing our results [12].

1.3 Bose-Einstein correlations

Our main goal is the investigation of the BEC, in particular whether these correlations show indeed some special features which could be attributed to the branchings and to their space-time and momentum space structure. We are therefore interested in two particle correlation function

$$C_2(Q = |p_i - p_j|) = \frac{dN(p_i, p_j)}{dN(p_i) dN(p_j)}. \quad (2)$$

To calculate it we have decided to use the ideas of the BEC “afterburners” advocated recently in [9]. Such step is necessary because cascade *per se* do not show bosonic bunching in momenta (as is the case in models where Bose statistics is incorporated from the beginning, like [13]; however, we cannot follow this strategy here). Because we are interested in some possible systematics of results rather than in particular values of the “radius” R and “coherence” λ parameters characterizing source M , we have chosen the simplest, classical version of such afterburner. After generating a set of $i = 1, \dots, N_l$ particles for the l^{th} event we choose all pairs of the same sign and endow them with the weight factors of the form

$$C = 1 + \cos[(r_i - r_j)(p_i - p_j)] \quad (3)$$

where $r_i = (t_i, \vec{r}_i)$ and $p_i = (E_i, \vec{p}_i)$ for a given particle. The signs are connected with charges which each cascade vertex is endowed with using simple rules: $\{0\} \rightarrow \{+\} + \{-\}$, $\{+\} \rightarrow \{+\} + \{0\}$ and $\{-\} \rightarrow \{0\} + \{-\}$.

2 Results

Although it is straightforward to get our cascade model in the form of the Monte Carlo code, the main features of $D = 1$ case can be also demonstrated analytically. For example, in the limiting cases of totally symmetric cascades (where for all vertices $k_{1,2} = k$), in which amount of energy allocated to the production is maximal, one gets the following multiplicity of produced particles:

$$N_s = 2^{L_{max}} = \left(\frac{M}{\mu}\right)^{d_F}, \quad d_F = \frac{\ln 2}{\ln \frac{1}{k}}. \quad (4)$$

It is entirely given by the length of the cascade, $L_{max} = \ln(M/\mu)/\ln(1/k)$, $\mu = \sqrt{m_0^2 + \langle p_T \rangle^2}$. The exponent d_F is formally nothing but a generalized (fractal) dimension of the fractal structure in phase space formed by our cascade. Notice the characteristic power-like behaviour of $N_s(M)$ in (4) which is normally attributed to thermal models. For example, for $k = 1/4$ one has $N_s \sim M^{1/2}$, which in thermal models would correspond to the ideal gas equation of state with velocity of sound $c_0 = 1/\sqrt{3}$ [5]. In the opposite limiting case of maximally asymmetric cascades, $M \rightarrow \mu + M_1$ (where $k_1 = \mu/M$ and $k_2 = k$) in which the amount of kinetic energy allocated to the produced secondaries is maximal, the corresponding multiplicity is equal

$$N_a = 1 + L_{max} = 1 + \frac{1}{\ln \frac{1}{k}} \cdot \ln \frac{M}{\mu}. \quad (5)$$

The dependence on L_{max} is now linear (i.e., dependence on the energy is logarithmic). The important feature, which turns out to be valid also in general, is the observed scaling in the ratio of the available mass of the source M and the mass of produced secondaries: M/μ .

The $D = 3$ case differs only in that the decay products can flow in all possible directions, which are chosen randomly from the isotropic angular distribution. To allow for some nonzero transverse momentum in $D = 1$, we are using the transverse mass $\mu = 0.3$ GeV. For $D = 3$ cascade μ is instead put equal to the pion mass, $\mu = 0.14$ GeV. All decays are described in the rest frame of the corresponding parent mass M_i in a given vertex. To get the final distributions one has to perform a necessary number of Lorentz transformations to the rest frame of the initial source mass M . As an output we are getting in each run (event) a number N_j of secondaries of mass μ with energy-momenta $(E_j; \vec{P}_j)_i$ and birth space-time coordinates $(t_j; \vec{r}_j)_i$, $i = 1, \dots, N_j$ (i.e., coordinates of

the last branching). Results presented in Figs. 1 and 2 are obtained from 50000 such events. Decay parameters $k_{1,2}$ were chosen randomly from a triangle distribution $P(k) = (1 - k)$ (leading to a commonly accepted energy behaviour of $N(M) \sim M^{0.4 \div 0.5}$, as discussed above). For more detailed presentation of rapidity and multiplicity distributions and demonstration of intermittent behaviour of factorial moments see [5].

Fig. 1 shows densities $\rho(r)$ of points of production and correlation function $C_2(Q)$ as defined in (3) for $D = 1$ and 3 dimensional cascades originated from masses $M = 10, 40$ and 100 GeV. The evolution parameter is set equal $\tau = 0.2$ fm (in [5] we discuss also $\tau \sim 1/M$ case). The decay function Γ is taken exponential (i.e., $q = 1$). We observe, as expected in [3], a power-like behaviour of cascading source:

$$\rho(r) \sim \left(\frac{1}{r}\right)^L \quad r > r_0, \quad (6)$$

but *only* for $r > r_0$, i.e., for radii larger then some (not sharply defined) radius r_0 , value of which depends on all parameters used: mass M of the source, dimensionality D and evolution parameter τ of the cascade. Below r_0 the $\rho(r)$ is considerably bended, remaining almost flat for $D = 1$ cascades. For the limiting case of $M = 100$ GeV the corresponding values of parameter L vary from $L = 1.89$ for one $D = 1$ cascades to $L = 2.78$ for $D = 3$ cascades. The shapes of $\rho(r)$ scale in the ratio M/μ in the same way as the multiplicities discussed before (the same remains also true for rapidity and multiplicity distributions, cf. [5]). One can summarize this part by saying that power-like behaviour [3] sets in (albeit only approximately) only for long cascades (large values of M/μ). It remains therefore to be checked whether (and to what extend) such conditions are indeed met in the usual hadronic processes.

The corresponding BEC functions C_2 show a substantial differences between $D = 1$ and $D = 3$ dimensional cascades, both in their widths and shapes. Whereas the former are more exponential-like (except, perhaps, for small masses M) the latter are more gaussian-like with a noticeably tendency to flattening out at very small values of Q observed for small masses M . Also values of intercepts, $C_2(Q = 0)$, are noticeable lower for $D = 3$ cascades. The length of the cascade (i.e., the radius of the production region, cf. discussion of density ρ before) dictates the width of $C_2(Q)$. However, the M/μ scaling observed before in shapes of source functions is lost here. This is because C_2 depends on the differences of the momenta $p = \mu \cosh y$, which do not scale in M/μ . The flattening mentioned above for $D = 3$ cascades are the most distinctive signature of the fractal structure combined with $D = 3$ dimensionality of the cascade. The correlations of the position-momentum type existing here as in all flow phenomena are, in the case of $D = 3$ cascades, not necessarily vanishing for very small differences in positions or momenta between particles under consideration. The reason is that our space-time structure of the process can have in $D = 3$ a kind of “holes”, i.e., regions in which the number of produced particles is very small. This is perhaps the most characteristic observation for fractal (i.e., cascade) processes of the type considered here. This feature seems to be more pronounced for diluted cascades corresponding to $q < 1$ case discussed below.

Fig. 2 displays the same quantities but this time calculated for $M = 40$ GeV and for different values of parameter q in the decay function Γ defined in (1). This function is written there in form of Tsallis distribution [12], which allows to account for a variety of possible influences caused by, for example, long-range correlations, memory effects or for the possible additional fractal structure present in the production process. They all result in a non-extensivity of some thermodynamical variables (like entropy) with $|1 - q|$ being the measure of this non-extensivity. In practical terms

of interest here, for $q < 1$ the tail of distribution (1) is depleted and its range is limited to $t \in (0, \tau/(1-q))$ whereas for $q > 1$ it is enhanced in respect to the standard exponential decay law (and its range is $t \in (0, \infty)$) [12]. In other words, one can account in this way for both more diluted (for $q < 1$) and more condensed (for $q > 1$) space-time structure of the developing cascades. Such distributions are ubiquitous in numerous phenomena and they are founded in the, so called, Tsallis non-extensive thermostatics [12] generalizing the conventional Boltzmann-Gibbs one (which in this notation corresponds to the $q = 1$ case). It has found also applications in high energy and nuclear physics (cf. [11] for references). Its effect on the cascades investigated here is, as can be seen from Fig. 2, in that it mimics (to some extent) the changes attributed in Fig. 1 to different energies (making cascade effectively shorter for $q = 0.8$ and longer for $q = 1.2$). The results of Fig. 2 (taken for $M = 40$ GeV) should be then compared with those of Fig. 1 for $M = 10$ and $M = 100$ GeV. They demonstrate that effects of longer or shorter cascades in momentum space (as given by different M in Fig. 1) is similar to effects of the more or less condensed cascades in the position space as given by q here. This fact should be always kept in mind in such analysis as ours.

3 Conclusions

We conclude that BEC are, indeed, substantially influenced by the fact that the production process is of the cascade type (both in momentum and space-time) as was anticipated in [3], although probably not to the extent expected (which, however, has not been quantified there). In practical applications (fitting of experimental data) there are many points which need further clarification. The most important is the fact that data are usually collected for a range of masses M and among directly produced particles are also resonances. This will directly affect lengths of the cascades, and through them the final results for C_2 . Selecting events with similar masses M should allow to check whether in such processes $q = 1$ or not. The importance of this finding is in that $q < 1$ would signal a non-stochastic development of the cascade, whereas $q > 1$ would indicate that, as has been discussed in [11], parameter τ fluctuates with relative variance

$$\omega = \frac{\left\langle \left(\frac{1}{\tau}\right)^2 \right\rangle - \left\langle \frac{1}{\tau} \right\rangle^2}{\left\langle \frac{1}{\tau} \right\rangle^2} = q - 1. \quad (7)$$

Such fluctuations are changing exponential behaviour of Γ in (1) to a power-like distribution with enhanced tail.

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Figure Captions

Fig. 1. Density distribution of the production points $\rho(r)$ (left panels) and the corresponding $C_2(Q = |p_i - p_j|)$ (right panels) for one-dimensional (upper panels) and three-dimensional (lower panels) cascades. Each panel shows results for $M = 10, 40$ and 100 GeV masses of the source. Time evolution parameter is $\tau = 0.2$ fm and nonextensivity parameter $q = 1$.

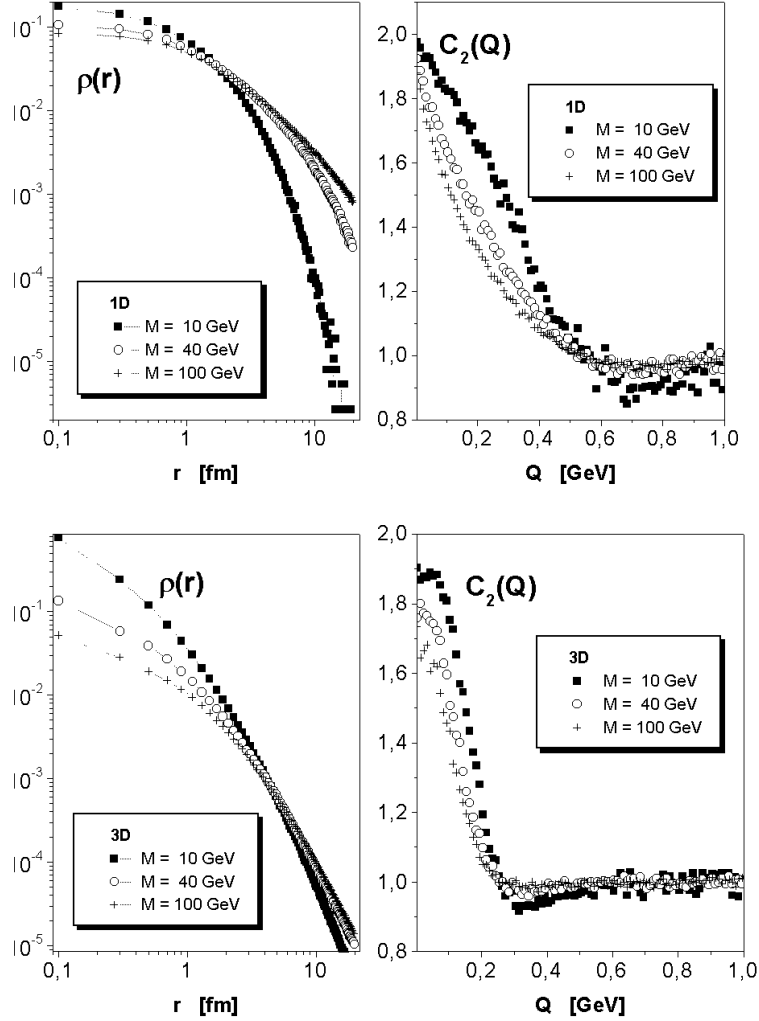


Fig. 2. The same as in Fig. 1 except that each panel shows results for mass of the source $M = 40$ GeV and for three different values of the nonextensivity parameter $q = 0.8, 1.0$ and 1.2 .

